Autonomous Instruments and Feature Extractor Feedback Systems

Risto Holopainen

Department of Musicology

15 june 2012

Autonomous instruments

- Generate music by algorithms from the waveform to an entire composition
- No realtime interaction
- Essentially a deterministic autonomous system

Bytebeat

One-liners:

```
int t;
for(t=0; t<T; t++)
 putchar( expression );
```

Sawtooth wave:

```
3.5*t
```

Periodic pattern:

```
5*t ^ 3*(t + t%12) | (t/125)
```

Analog feedback systems as autonomous instruments

Patch without feedback
Same with feedback

Dynamic systems

Consider a map $f : \mathbb{R}^d \to \mathbb{R}^d$.
Generate a time series $\mathbf{x}_n$ by iteration:

$$\mathbf{x}_{n+1} = f(\mathbf{x}_n) \quad \text{or} \quad x_n = f^n(\mathbf{x}_0)$$

Use $\mathbf{x}_n$ as audio samples.

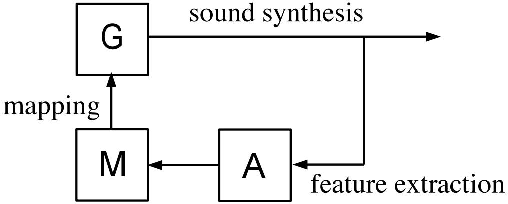
Qualitative dynamics:

- Fixed point: $x_{n+1} = x_n$
- Periodic: $x_{n+T} = x_n$
- Quasi-periodic
- Unbounded: $\lim_{n\to\infty} \|x_n\| = \infty$
- Chaotic

Feature extractor feedback systems (FEFS)

Basic ingredients:

- Oscillator (signal generator) **G**
- Feature extractor / analysis **A**
- Mapping **M**



The FEFS equation

$$
\begin{aligned}
x_n &= \mathbf{G}(\pi_n, n) \\
\phi_n &= \mathbf{A}(x_n, x_{n-1}, \ldots, x_{n-L+1}) \\
\pi_{n+1} &= \mathbf{M}(\phi_n, \pi_n)
\end{aligned}
$$

where
π are the synthesis parameters
ϕ is the feature extractor signal

Feature extraction

Low level feature extractors:

- RMS amplitude

$$RMS(x_n) = \left\langle x_n^2 \right\rangle^{1/2}$$

- Spectral centroid

$$Centroid(x_n) = \frac{RMS(\frac{d}{d_n}(x_n))}{RMS(x_n)}$$

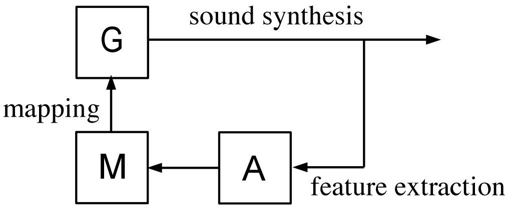
The extended time window

$$\phi_n = \mathbf{A}(x_n, x_{n-1}, \dots, x_{n-L+1})$$

causes smoothing of x_n.
Simplified structure of the FEFS equation:

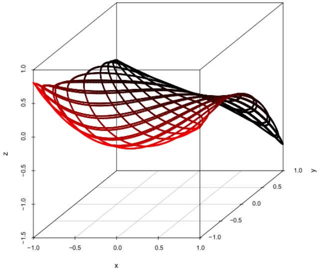
$$x_n = \mathbf{F}(x_{n-1}, x_{n-2}, \dots, x_{n-L})$$

FEFS example



G: Wave terrain synthesis
A: amplitude + centroid
M: complicated mapping

Wave terrain synthesis



$$
\begin{aligned}
z_n &= f_\mu(x_n, y_n) \\
x_n &= A_x \cos(\omega_x n) \\
y_n &= A_y \cos(\omega_y n)
\end{aligned}
$$

where $f_\mu(x, y) = \mu_1 x_n y_n + \mu_2(x_n^2 - y_n^2) + \mu_3(x_n + y_n)^3$

Seven free parameters: $\pi = (A_x, A_y, \omega_x, \omega_y, \mu_1, \mu_2 \mu_3)$

The wave terrain system

Send $z_n = f_\mu(x_n, y_n)$ to feature extractors

$$\begin{aligned}\phi_1(z_n) &\equiv RMS(z_n)\\ \phi_2(z_n) &\equiv Centroid(z_n)\end{aligned}$$

Map the features to synthesis parameters with $\mathbf{M} : \mathbb{R}^2 \rightarrow \mathbb{R}^7$,

$$\pi_{n+1} = f(\phi_1(z_n), \phi_2(z_n))$$

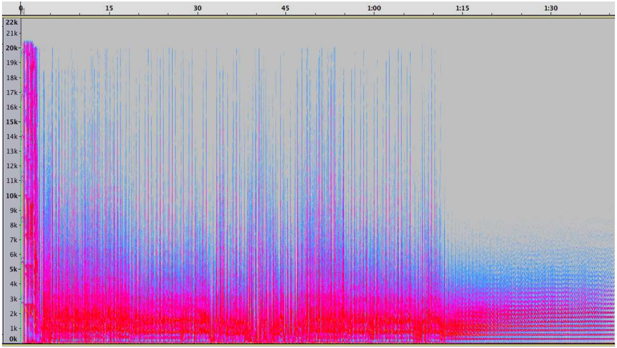
The mapping

```
void mapp::map(double AA, double CC, params &pp)
{
// amp AA, centroid CC --> params. pp; internal coefs

    pp.a = a[0] + a[1]*CC + a[2]*AA + a[3]*CC*CC + a[4]*AA*AA + a[5]*CC*AA;
    pp.b = b[0] + b[1]*CC + b[2]*AA + b[3]*CC*CC + b[4]*AA*AA + b[5]*CC*AA;
    pp.c = c[0] + c[1]*CC + c[2]*AA + c[3]*CC*CC + c[4]*AA*AA + c[5]*CC*AA;

    pp.A = A[0] + A[1]*CC + A[2]*AA + A[3]*cos(A[5]*CC + A[6]) + A[4]*cos(A[7]*AA + A[8]);
    pp.B = B[0] + B[1]*CC + B[2]*AA + B[3]*cos(B[5]*CC + B[6]) + B[4]*cos(B[7]*AA + B[8]);

    pp.f1 = F[0] + F[1]*CC + F[2]*AA + F[3]*cos(F[5]*CC + F[6]) + F[4]*cos(F[7]*AA + F[8]);
    pp.f2 = G[0] + G[1]*CC + G[2]*AA + G[3]*cos(G[5]*CC + G[6]) + G[4]*cos(G[7]*AA + G[8]);

}
```

54 new constants for the mapping!

The halting problem

Very long chaotic transients
or stable chaos that goes on forever?

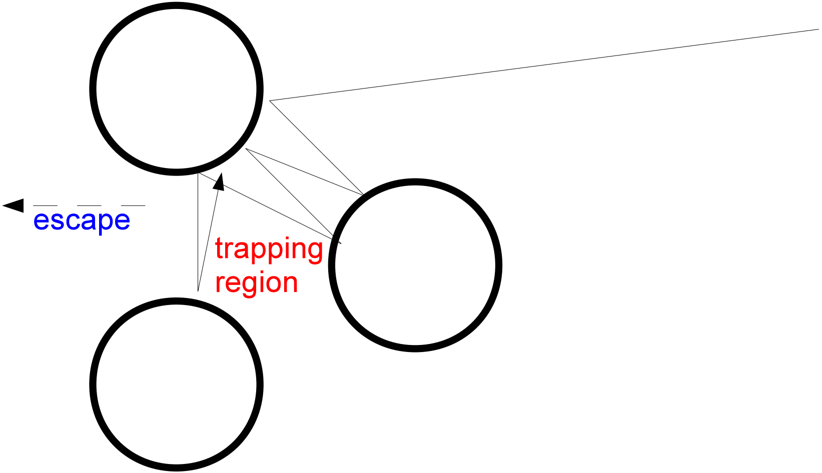


Repetition detector

$$\begin{aligned} \text{if}(|a_n - a_{n-d}| &< \epsilon \\ \& \ |a_{n-d} - a_{n-2d}| &< \epsilon \\ \& \ |c_n - c_{n-d}| &< \epsilon \\ \& \ |c_{n-d} - c_{n-2d}| &< \epsilon) \end{aligned}$$

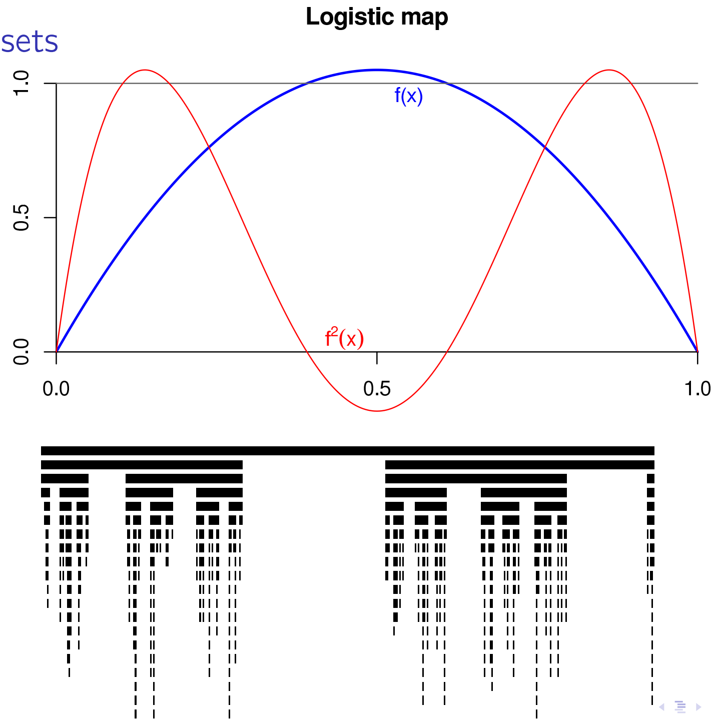
then halt!

Chaotic scattering



Chaotic transients (pinball game)

Cantor sets



Halting times